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Dense stellar matter with strangeness: Kaon condensation and strange quark matter

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Summary. — The density of compact star (neutron-star) core is supposed to be much higher than the normal nuclear-matter density. Among the various possibilities the emergence of strangeness at higher density has been suggested, in the form of, for example, meson or hyperon condensation and/or deconfined quark matter. In this work, we explore the possible effect of strangeness on the nuclear symmetry energy, which is responsible for how new degrees of freedom can be populated in the nucleon matter. And we discuss the scenario where the kaon condensed matter is driving the system into a strange quark matter.

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PACS 21.65.-f – Nuclear matter.
PACS 06.30.Dr – Mass and density.

1. – Introduction

The new degrees of freedom other than nucleon such as meson condensations (pions, kaons) and/or hyperons, quark matter with perturbative and/or nonperturbative variations, have been discussed as possible constituents of the core of compact stars. The recent observation of a 1.97 solar-mass ($M_\odot$) neutron star, PSR J1614-2230 [1], raises a highly pertinent issue on whether such non-nuclear degrees of freedom are relevant for the physics of stable compact stars.

One of the examples of meson condensations is the scenario of Bethe and Brown [2,3], where the onset of kaon condensation [4] inside the neutron star matter at a density $\rho \sim 3\rho_0$—where ρ_0 is the nuclear matter density—keeps the maximum mass less than $2M_\odot$, which seems to be consistent with the observations of well-measured neutron star masses $\sim 1.5M_\odot$ [5]. If deconfinement occurs at the core, the strange quark matter suggested by Witten [6] is a possible candidate.

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Recently an interesting scenario [7] has been developed in which the compact star has a triple-layered structure, from outside to the core, normal nuclear matter, kaon condensed matter, and quark matter. Such a triple-layered compact star can be made compatible with PSR J1614-2230 by adjusting the minimal number of parameters of the model.

To introduce new degrees of freedom into the system, there should be a mechanism that makes the matter to relax the neutron-proton asymmetry present, by which the system will evolve to a matter in a more stable state. Nuclear symmetry energy is a measure how the relaxation of asymmetry costs.

However the change of neutron and proton fractions of the star matter would require the isospin to be violated, which can happen via the weak interactions, where the flavor (isospin or strangeness) can be changed. We assume that the evolution of the system via weak interactions reaches an equilibrium configuration as a ground state of the matter. Therefore, the EoS of the star matter in weak equilibrium must be strongly dependent on the nuclear symmetry energy. New degrees of freedom we are considering involve electron, muon, kaon, hyperon, and strange-quark matter (SQM).

In sect. **2**, the nuclear symmetry energy is introduced with the discussion on the effect of the presence of strange hadrons in the matter. The basic features of kaon condensation driving the system into SQM is discussed using a simplified model for kaon condensation and nucleon-nucleon interaction in sect. **3**. The summary is given in sect. **4**

2. – Nuclear symmetry energy

2˙1. *Isospin symmetry* [8]. – The (strong) interaction between nucleons is charge symmetric and has been formulated in an $SU(2)$ symmetric way, *i.e.*, isospin symmetry. Proton and neutron are assigned to be members of a doublet and all hadrons are classified into $SU(2)$ multiplets. The interaction Hamiltonian commutes with $SU(2)$ generators, $\vec{I}$,

$$[I_i, H_{int}] = 0, \tag{1}$$

and the proton ($I_3 = 1/2$) numbers and neutron ($I_3 = -1/2$) numbers are conserved, so that we can classify the eigenstate by the definite number of protons and neutrons ($I_3 = 1/2(N_p - N_n)$):

$$|N_p, N_n\rangle. \tag{2}$$

The energy of the eigenstate of the Hamiltonian does not depend on I_3 but on I^2. The eigenstate eq. (2), can be decomposed into the irreducible representations (multiplets) of $SU(2)$, Clebsch-Gordan series, as

$$|N_p, N_n\rangle = \Sigma_I C_I |I : N_p, N_n\rangle, \tag{3}$$

where $|\vec{I}|^2 = I(I+1)$. The energy of the state is given by

$$E(N_p, N_n) = \langle N_p, N_n|H|N_p, N_n\rangle = \Sigma_I |C_I|^2 E_I, \tag{4}$$

where E_I is a reduced matrix element of the Hamiltonian, $H = H_0 + H_{int}$,

$$E_I = \langle I||H||I\rangle, \tag{5}$$

which is independent of I_3, and depends on the details of the strong interactions for each I-channel. Although it may appear that the energy is independent of the compositions of protons and neutrons for a given total number of nucleons

$$N = N_p + N_n, \tag{6}$$

different compositions have different decompositions into multipletes, *i.e.*, different sets of C_I. Therefore different compositions of protons and neutrons leads to different energies. This explains why the nuclear symmetry energy appears in asymmetric nuclear matter. It is not a result of isospin symmetry breaking of the strong interactions. The strong interaction is isospin-symmetric, but we are considering the states with different Clebsch-Gordan decompositions.

For a given number of proton number fraction $x(= \rho_p/\rho)$, C_I and E_I are function of x and density ρ: $C_I(\rho, x)$ and $E_I(\rho, x)$. Then for nuclear matter in infinite system with density n the energy per nucleon can be decomposed as

$$E(\rho, x) = E(\rho, x = 1/2) + E_{sym}(\rho, x), \tag{7}$$

for a neutral system. E_{sym} measures the iso-spin dependent part of the energy with respect to the $n - p$ symmetric matter ($x = 1/2$). Since the interaction itself is iso-symmetric, naive expectation is that the $n - p$ symmetric system is the lowest configuration, and we can approximate the symmetry energy around $x = 1/2$ as

$$E_{sym}(\rho, x) = (1 - 2x)^2 S(\rho), \tag{8}$$

where $S(\rho)$ is called symmetry energy factor(or simply symmetry energy). The absence of linear term in $(1 - 2x)$ is due to the permutation symmetry of n and p.

Whether eq. (8) is also valid up to $x = 0$ or 1 depends on the details of nuclear interaction. One simple example where eq. (8) is a very good approximation for all x ranging from 0 to 1 is free nucleon gas, which are subjected only to Pauli exclusion principle. A straightforward calculation for the non relativistic nucleon gas shows that it is a very good approximation for $x = 0 - 1$ and it gives

$$S^{free}(\rho) = \left(2^{2/3} - 1\right) \frac{3}{5} E_F^0 \left(\frac{\rho}{\rho_0}\right)^{2/3}, \tag{9}$$

where $E_F^0 = \frac{(3\pi^2\rho_0/2)^{2/3}}{2m}$ is the Fermi energy at $\rho = \rho_0/2$.

Suppose we can decompose symmetry energy into two parts, $E_{sym}^{(kin)}$ and $E_{sym}^{(pot)}$,

$$E_{sym}(\rho, x) = E_{sym}^{(kin)}(\rho, x) + E_{sym}^{(pot)}(\rho, x). \tag{10}$$

One can easily guess that kinetic part can be approximated as in free nucleon system

$$E_{sym}^{kin}(\rho, x) = (1 - 2x)^2 S^{(kin)}(\rho). \tag{11}$$

$S^{(kin)}(\rho)$ can be very much different from the free case due to the change of fermi gas structure by nucleon interaction. Recent analysis [9] demonstrates that the correlation between higher momentum nucleons reduce the strength of symmetry energy factor to be much weaker than the potential part even at normal nuclear density.

However it is not well understood how the symmetry energy behaves particularly at higher density than normal nuclear density, ρ_0. There are two issues for $E_{sym}^{(pot)}$.

The first one is whether one can make use of the following form for the full range of $x = 0 - 1$,

$$E_{sym}^{(pot)}(\rho, x) = (1 - 2x)^2 S^{(pot)}(\rho) \tag{12}$$

as a good approximation at least. It should be noted that, in recent analysis [10], it is demonstrated that it is considered to be a good approximation. However we may not expect eq. (12) is valid when new degree of freedom, when strange hadrons are present in the system. The more discussion will be given in later section. The second question is then how $S^{(pot)}(\rho)$ changes with the density: whether it keeps growing or it is saturated or its slope changes from positive to negative or it is decreasing even to negative value at high enough density [11].

2˙2. *Clebsch-Gordan series with hyperon.* – In addition to isospin symmetry, $SU(2)$, under which the proton ($I_3 = 1/2$) numbers and neutron ($I_3 = -1/2$) numbers are conserved, there is a hypercharge $Y = S + B$(or strangeness numbers S) which is conserved in strong interaction. One can make use of $SU(3)$ for classification of hadrons, although it is broken explicitly by the heavy strange quark mass. The full symmetry is useful only when the correction by symmetry breaking is taken into account. In this work, we do not use the full $SU(3)$ symmetry but $SU(2)$ symmetry with hypercharge conservation. Now we can classify the eigenstate by the definite number of protons and neutrons and total number of hypercharge ($I_3 = 1/2(N_p - N_n), Y = \Sigma_i Y_i N_{(Y_i)}$):

$$|N_p, N_n, Y\rangle. \tag{13}$$

Since isospin symmetry is not broken, the energy of the eigenstate of the Hamiltonian depends only on I^2 and additionally on Y. However since $SU(3)$ symmetry is broken explicitly, it is better to have the eigenstate, eq. (13), decomposed into the irreducible representations (multiplets) of $SU(2)$, as given by

$$|N_p, N_n, Y\rangle = \Sigma_{(I,Y)} C_{(I,Y)} |I, Y : N_p, N_n\rangle, \tag{14}$$

where $|\vec{I}\,|^2 = I(I+1)$. The energy of the state is given by

$$E(N_p, N_n, Y) = \langle N_p, N_n, Y|H|N_p, N_n, Y\rangle = \Sigma_{(I,Y)} |C_{(I,Y)}|^2 E_{(I,Y)}, \tag{15}$$

where $E_{(I,Y)}$ is an reduced matrix element of the Hamiltonian, $H = H_0 + H_{int}$,

$$E_{(I,Y)} = \langle I, Y||H||I, Y\rangle, \tag{16}$$

which is independent of I_3, and depends on the details of the strong interactions for each (I, Y)-channel.

Therefore one can expect the nuclear symmetry energy can be affected by the presence of hyperon. The simplest parametrization inferred from the pure nuclear matter is

$$E(\rho, x, y_i) = E(\rho, x = 1/2, y_i) + (1 - 2x)^2 E_{sym}(\rho_N, y_i), \tag{17}$$

where y_i are the density of hyperon i and ρ_N is the nucleon number density $\rho_N = \rho_n + \rho_p$. ρ is the total baryon number density $\rho = \rho_N + y$, where $y = \Sigma_i y_i = n - \rho_N$ is the total hyperon number density.

Whether the naive extension of eq. (17) with $(1-2x)^2$ dependence is valid or not should be verified theoretically and tested experimentally. For the physical processes in which the hyperons are not appearing isosymmetrically due to energy barrier or n-p asymmetry present in nuclear matter, eq. (17) may not be valid. Another question to be asked is whether we can make use a symmetry energy determined or calculated in pure nucleon matter in the presence of hyperon such as

$$E_{sym}(\rho_N, y_i) \approx E_{sym}(\rho_N, y_i = 0). \tag{18}$$

So far it remains an open question, which will be discussed elsewhere. Through out this work we adopt eq. (18) as an approximation to begin with.

3. – Kaon Condensation and strange-quark matter

The key ingredient for kaon condensation [4] is the decrease of the effective mass denoted as m_K^* of the negatively charged kaon K^- as density increases. The m_K^* is basically a function of m_K, ρ_n, ρ_p owing to the kaon-nucleon interactions,

$$m_K^* = \omega(m_K, \rho_n, \rho_p, \ldots). \tag{19}$$

The density at which a neutron can decay into a proton and K^- via the weak process, $n \to p + K^-$,

$$\mu_n - \mu_p = m_K^*, \tag{20}$$

determines the threshold density of kaon condensation. Above the kaon condensation, where m_K^* can be identified as the kaon chemical potential μ_K, the chemical equilibrium is reached as

$$\mu_n - \mu_p = \mu_e = \mu_\mu = \mu_K \equiv \mu, \tag{21}$$

where

$$\mu_n - \mu_p = 4(1-2x)S(\rho) + \Theta(K)F(K,\mu). \tag{22}$$

K stands for the kaon amplitude of kaon condensed state, *i.e.*, $\langle K \rangle$, and $F(K,\mu)$ is a nontrivial function that depends on the neutron-proton chemical potential difference which, in turn, depends on kaon-nucleon interactions. The charge neutrality condition gives

$$\rho_p = \rho_e + \rho_\mu + \Theta(K)\rho_K. \tag{23}$$

Equations (19), (22), and (23) are the basic equations to be solved to calculate the EoS of kaon condensed nuclear matter (KNM).

Strange quark matter can appear as a result of confinement-deconfinement phase transition. At the phase boundary with critical density, ρ_c, the chemical equilibrium reads

$$\mu_n = 2\mu_d + \mu_u, \qquad \mu_p = \mu_d + 2\mu_u. \tag{24}$$

We suppose the confinement-deconfinement phase transition taking place constrained by the weak equilibrium in the kaon condensed matter, leading to SQM. When the kaon chemical potential—equivalently effective mass—μ as well as $F(K,\mu)$ approache 0 at the critical density, the solution $x = \frac{\rho_p}{\rho} = 1/2$ for $n-p$ symmetric matter appears naturally at the phase boundary. Below we demonstrate it can be the case using the simplest form of an effective chiral Lagrangian [12] given by

$$\mathcal{L} = \mathcal{L}_{KN} + \mathcal{L}_{NN}, \tag{25}$$

where

$$\begin{aligned} \mathcal{L}_{KN} &= \partial_\mu K^- \partial^\mu K^+ - m_K^2 K^+ K^- + \frac{1}{f^2}\Sigma_{KN}(n^\dagger n + p^\dagger p)K^+K^- \\ &\quad + \frac{i}{4f^2}(n^\dagger n + 2p^\dagger p)(K^+\partial_0 K^- - K^-\partial_0 K^+), \end{aligned} \tag{26}$$

$$\mathcal{L}_{NN} = n^\dagger i\partial_0 n + p^\dagger i\partial_0 p - \frac{1}{2m}(\vec{\nabla} n^\dagger \cdot \vec{\nabla} n + \vec{\nabla} p^\dagger \cdot \vec{\nabla} p) - V_{NN}. \tag{27}$$

Here the fourth term in eq. (26) is the well-known Weinberg-Tomozawa (WT) term which is constrained by a low-energy theorem with f identified with the pion decay constant f_π in the *matter-free* space and Σ_{KN} is the KN sigma term, which we take as one of parameters.

For s-wave kaon condensation, kaon amplitude, K, and kaon chemical potential, $\mu_K = \mu$, are defined by the ansatz

$$K^\pm = K e^{\pm i\mu t}. \tag{28}$$

Then we get

$$F(K,\mu) = \frac{\mu}{2f^2}K^2. \tag{29}$$

The kaon condensation condition for $K \neq 0$ is obtained by extremizing the classical action,

$$m_K^2 - \mu^2 = \mu\frac{\rho_n + 2\rho_p}{2f^2} + \frac{\rho}{f^2}\Sigma_{KN}, \tag{30}$$

which can be solved to get μ or equivalently m_K^*. The density dependence of μ is shown in fig. 1 for $\Sigma_{KN} = 200, 300, 400\,\mathrm{MeV}$ using CKL model [13]. One can see that μ or equivalently the in-medium effective kaon mass m_K^* vanishes at some high density, which

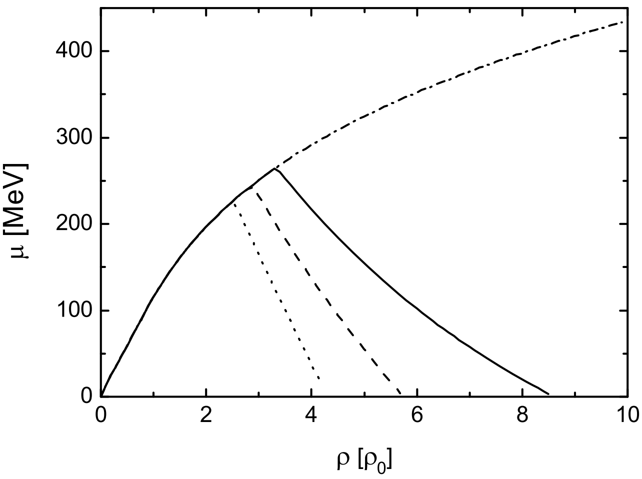

Fig. 1. – The density dependence of the chemical potential using a CKL model [13] with $\eta = -1$. The solid, dashed, and dotted lines correspond, respectively, to $\Sigma_{KN} = 200, 300, 400\,\mathrm{MeV}$ [7].

we refer to as critical density denoted ρ_c, as a phase boundary for deconfinement. Hence $F(K,\mu)$ in eq. (29) vanishes simultaneously. Then eq. (22) tells us that $x = 1/2$ is a natural solution and the system is driven to $n-p$ symmetric matter($x = 1/2$) by kaon condensation.

At the phase boundary, the chemical equilibrium (via confinement-deconfinement) reads

$$\mu_n - \mu_p = \mu_d - \mu_u, \quad \mu_{K^-} = \mu_s - \mu_u. \tag{31}$$

Since $\mu(=\mu_K) = 0$, we have from eq. (31)

$$\mu_u = \mu_d = \mu_s. \tag{32}$$

This is the chemical potential relation for the SQM in the masselss limit. In this simple picture, the KNM leads naturally to a SQM at the critical density ρ_c defined by the condition, $\mu_K = 0$. Equation (32) implies that they have the same number densities, $\rho_u = \rho_d = \rho_s = \rho_Q$. Then the charge neutrality

$$\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s = 0 \tag{33}$$

is automatically satisfied and there is no need for additional leptons. Kaon condensed nuclear matter will naturally go over to the SQM in $SU(3)$ symmetric phase in the massless limit.

The EoS of SQM is then given by

$$\epsilon_{SQM} = 4.83 a_4 \rho^{4/3} + B, \tag{34}$$

$$P_{SQM} = 1.61 a_4 \rho^{4/3} - B, \tag{35}$$

where B is the bag constant [14]. Here a_4 denotes the perturbative QCD correction [15, 16], which takes the value $a_4 \le 1$. The equality holds for SQM without QCD corrections.

The energy density and the pressure of the kaon condensed nuclear matter (KNM) are given by

$$\epsilon_{KNM} = \tilde{V}(\rho) + \rho(1-2x)^2 S(\rho) + \epsilon_{lepton} + \Theta(K)\epsilon_K, \tag{36}$$

$$P_{KNM} = \rho^2 \frac{\partial V(\rho)/\rho}{\partial \rho} + \rho^2(1-2x)^2 \frac{\partial S(\rho)}{\partial \rho} + P_{lepton} + \Theta(K) P_K. \tag{37}$$

For $V(\rho)$ and $S(\rho)$, we make use a parametrization suggested by Li *et al.* [13] with $\eta = -1$. The contributions to the energy density and the pressure from kaon condensation are given by

$$\epsilon_K = \left(m_K^2 + \mu_K^2 - \frac{\rho}{f^2}\Sigma_{KN} \right) K^2, \tag{38}$$

$$P_K = -\left(m_K^2 - \mu^2\right) K^2. \tag{39}$$

One can see that the kaon condensation gives a negative contribution to the total pressure for $\mu < m_K$.

The pressure matching condition at the boundary,

$$P_{KNM}(\rho_c) = P_{SQM}(\rho_c^Q), \tag{40}$$

can be solved to find a set of parameters, $a_4, B, \Sigma_{KN}, V(\rho)$, and $S(\rho)$. Then it is possible to consider a triple-layered structure consisting of NM, KNM, and SQM from the outer layer to the core part. The resulting mass-radius relation which is compatible with PSR has been discussed in [7]. It is worth mentioning that the refined analysis, which will be discussed elsewhere [17], gives maximum mass of $1.99\, M_\odot$ and radius of 11.12 km with core density of $11.7\ \rho_0$ for a model of $\eta = -1$ with $B^{1/4} \simeq 101\,\text{MeV}$, $\Sigma_{KN} \simeq 260\,\text{MeV}$, and $a_4 = 0.624$.

4. – Summary

We discussed the basic nature os nuclear symmetry energy and the issues for the symmetry energy in the presence of strange hadrons, strange mesons and hyperons. The role of symmetry energy for new degrees of freedom in the nuclear matter is also sketched in connection to the kaon condensation. We discussed the underlying mechanism for a scenario in which dense compact-star matter is driven smoothly to an SQM by kaon condensation at the density at which the kaon chemical potential $\mu_K = m_K^*$ becomes negligibly small and at which the nuclear matter becomes n-p symmetric, $x = 1/2$. It leads to a suggestion of new possible scenario for the compact star with an NM-KNM-SQM structure, which is found to be consistent with recently observed high mass neutron star of $1.97 M_\odot$ by using the parameters that are not excluded by theory or phenomenology.

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REFERENCES

[1] Demorest P. B. *et al.*, *Nature*, **467** (2010) 1081.
[2] Bethe H. A. and Brown G. E., *Astrophys. J.*, **445** (1995) L129; **423** (1994) 659.
[3] For a review, Brown G. E., Lee C.-H. and Rho M., *Phys. Rep.*, **462** (2007) 1.
[4] Kaplan D. B. and Nelson A. E., *Phys. Lett. B*, **175** (1986) 57.
[5] Thorsett S. E. and Chakrabarty D., *Astrophys. J.*, **512** (1999) 288; Lyne A. G. *et al.*, *Science*, **303** (2004) 1153; Faulkner A. J. *et al.*, *Astrophys. J.*, **618** (2004) L119.
[6] Witten E., *Phys. Rev. D*, **30** (1984) 272; Fahri E. and Jaffe R. L., *Phys. Rev. D*, **30** (1984) 2379.
[7] Kim K., Lee H. K. and Rho M., *Phys. Rev. C*, **84** (2011) 035810.
[8] Lee H. K. and Rho M., "Flavor symmetry and nuclear symmetry energy" arXiv:1201.6486.
[9] See for example, Carbone A., Polls A. and Rios A., arXiV:1111.0797.
[10] Dong H., Kuo T. T. S. and Machleidt R., *Phys. Rev. C*, **83** (2011) 054002.
[11] Wen D. H., Chen L.-W. and Li B.-A., *Phys. Rev. Lett.*, **103** (2009) 211102.
[12] We follow the line of discussion and the notation of K. Kubodera, *J. Korean Phys. Soc.*, **26** (1993) S171.
[13] Li B.-A., Chen L.-W. and Ko C. M., *Phys. Rep.*, **464** (2008) 113.
[14] Chodos A. *et al.*, *Phys. Rev. D*, **9** (1974) 3471.
[15] Lattimer J. M. and Prakash M., [arXiv:1012.3208 [astro-ph.SR]].
[16] Weissenborn S., Sagert I., Pagliara G., Hempel M. and Schaffner-Bielich J., [arXiv:1102.2869 [astro-ph.HE]].
[17] Kim K., Lee H. K. and Rho M., in preperation.